

Math 8 Honors Training on Divisibility Worksheet

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| 1. | <p>Problem 1. If k is a positive integer divisible by 9, and if $k < 200$, what is the greatest possible value of k?</p> <p>(A) 99 (B) 189 (C) 197 (D) 198 (E) 199</p> |
| 2. | <p>Problem 2. which of the following numbers can be used to show that the statement below is FALSE?</p> <p><u>All numbers that are divisible by both 6 and 9 are also divisible by 54.</u></p> <p>(A) 162 (B) 108 (C) 9 (D) 72 (E) 54</p> |
| 3. | <p>Problem 3. On a rectangular gameboard that is divided into n rows of m squares each, k of these squares not lie along the boundary of the gameboard. Which of the following is a possible value for k?</p> <p>(A) 15 (B) 25 (C) 35 (D) 49 (E) 52</p> |
| 4. | <p>Problem 4. When the positive integer s is divided 12, the remainder is 6. When the positive integer t is divided by 12, the remainder is 9. What is the remainder when the product st is divided by 9?</p> <p>(A) 1 (B) 3 (C) 5 (D) 7 (E) 0</p> |
| 5. | <p>Problem 5. If x is an integer and $3x$ is divisible by 15, which of the following must be true?</p> <p>I. x is divisible by 15. II. x is divisible by 5. III. x is an odd number.</p> <p>(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III</p> |
| 6. | <p>Problem 6. If x is divisible by 7 and y is divisible by 8. Which of the following must be divisible by 56?</p> <p>I. xy II. $7x + 8y$ III. $8x + 7y$</p> <p>(A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III</p> |
| 7 | <p>Problem 7. If a, b, c and d are different positive integers such that a is divisible by b, b is divisible by c, and c is divisible by d, which of the following statements must be true?</p> <p>I. a is divisible by cd. II. a has at least 4 positive factors. III. $a = bcd$</p> <p>(A) I only (B) II only (C) I and II (D) I and III only (E) I, II, and III</p> |

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| 8 | <p>Problem 8. The four-digit number $\underline{6BB5}$ is divisible by 25. How many such four-digit numbers are there?</p> <p>(A) 0 (B) 1 (C) 3 (D) 2 (E) 4</p> |
| 9 | <p>Problem 9. The five-digit number $\underline{31d26}$ is divisible by 3. Find the sum of all possible values of d.</p> <p>(A) 18 (B) 16 (C) 15 (D) 14 (E) 8</p> |
| 10 | <p>Problem 10. The three-digit number $\underline{6x4}$ is divisible by 7. What is the value of x?</p> <p>(A) 1 (B) 2 (C) 3 (D) 4 (E) 5</p> |
| 11 | <p>Problem 11. When Rachel divides her favorite number by 7, she gets a remainder of 5. What will the remainder be if she multiplies her favorite number by 5 and then divides by 7?</p> <p>(A) 4 (B) 3 (C) 2 (D) 1 (E) 0</p> |
| 12 | <p>Problem 12. For what digit n is the five-digit number $3n85n$ divisible by 6?</p> <p>(A) 0 (B) 1 (C) 2 (D) 3 (E) 4</p> |
| 13 | <p>Problem 13. What digit should replace the tens digit d so that the seven-digit number $5,376,5d4$ is divisible by 24?</p> <p>(A) 4 (B) 3 (C) 2 (D) 1 (E) 0</p> |
| 14 | <p>Problem 14. How many 2-digit numbers are not divisible by 13?</p> <p>(A) 90 (B) 83 (C) 13 (D) 7 (E) 84</p> |
| 15 | <p>Problem 15. How many numbers less than 100 and divisible by 3 are also divisible by 4?</p> <p>(A) 96 (B) 42 (C) 8 (D) 25 (E) 33</p> |
| 16 | <p>Problem 16. There are 24 four-digit numbers which use each of the digits 1, 2, 3, 4. How many of these are divisible by 11?</p> <p>(A) 10 (B) 6 (C) 5 (D) 4 (E) 8</p> |
| 17 | <p>Problem 17. Find a digit d that makes the three-digit number $\underline{2d6}$ a multiple of 22.</p> <p>(A) 1 (B) 4 (C) 5 (D) 8 (E) 2</p> |

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| 18 | <p>Problem 18. A four-digit number uses each of the digits 1, 2, 3 and 4 exactly once. What is the probability that the number is a multiple of 4?</p> <p>(A) 1/2 (B) 1/3 (C) 1/5 (D) 1/4 (E) 3/8</p> |
| 19 | <p>Problem 19. What is the greatest three-digit number that is divisible by 6?</p> <p>(A) 999 (B) 998 (C) 997 (D) 996 (E) 993</p> |
| 20 | <p>Problem 20. If the four-digit number <u>5,7d 2</u> is divisible by 18, what is d?</p> <p>(A) 4 (B) 3 (C) 2 (D) 1 (E) 0</p> |
| 21 | <p>Problem 21. What digit can replace K in the number <u>9K73K0</u> so that <u>9K73K0</u> will be divisible by 60?</p> <p>(A) 4 (B) 3 (C) 2 (D) 1 (E) 0</p> |
| 22 | <p>Problem 22. How many different 4-digit numbers can be formed using the digits 2, 4, 5, 6, and 7 such that no digits repeat and the number is divisible by 4?</p> <p>(A) 24 (B) 36 (C) 22 (D) 31 (E) 120</p> |
| 23 | <p>Problem 23. Given that m and n are digits, what is the sum of the values for m and n such that the five-digit number $m6,79n$ is divisible by 72?</p> <p>(A) 4 (B) 3 (C) 7 (D) 10 (E) 5</p> |
| 24 | <p>★Problem 24. (2011 AMC Problem 22) What is the tens digit of 7^{2011}?</p> <p>(A) 0 (B) 1 (C) 3 (D) 4 (E) 7</p> |
| 25 | <p>The 5-digit number <u>2 0 1 8 U</u> is divisible by 9. What is the remainder when this number is divided by 8?</p> <p>(A) 1 (B) 3 (C) 5 (D) 6 (E) 7</p> |
| 26 | <p>The 7-digit numbers <u>74A52B1</u> and <u>326AB4C</u> are each multiples of 3. Which of the following could be the value of C?</p> <p>(A) 1 (B) 2 (C) 3 (D) 5 (E) 8</p> |
| 27 | <p>Let "z" be a 6 digit positive integer such that 247,247, whose first three digits are the same as its last three digits taken in the same order. Which of the following numbers must also be a factor of "Z"</p> <p>a) 11 b) 19 c) 101 d) 111 e) 1111</p> |

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| 28 | <p>What is the sum of the distinct prime integer divisors of 2016?</p> <p>(A) 9 (B) 12 (C) 16 (D) 49 (E) 63</p> |
| 29 | <p>What is the largest power of 2 that is a divisor of $13^4 - 11^4$?</p> <p>(A) 8 (B) 16 (C) 32 (D) 64 (E) 128</p> |
| 30 | <p>Let N be the greatest five-digit number whose digits have a product of 120. What is the sum of the digits of N?</p> <p>(A) 15 (B) 16 (C) 17 (D) 18 (E) 20</p> |
| 31 | <p>The smallest number greater than 2 that leaves a remainder of 2 when divided by 3, 4, 5, or 6 lies between what numbers?</p> <p>(A) 40 and 50 (B) 51 and 55 (C) 56 and 60 (D) 61 and 65 (E) 66 and 99</p> |
| 32 | <p>The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number $PQRST$. The three-digit number PQR is divisible by 4, the three-digit number QRS is divisible by 5, and the three-digit number RST is divisible by 3. What is P?</p> <p>(A) 1 (B) 2 (C) 3 (D) 4 (E) 5</p> |
| 33 | <p>How many four digit positive integers (that is, integers between 1000 and 9999 inclusive) having only even digits are divisible by 5?</p> <p>a) 80 b) 100 c) 125 d) 200 e) 500</p> |
| 34 | <p>AMC 10B 2020) How many positive integers that are multiples of 3 and less than 2020 that are also perfect squares?</p> <p>a) 7 b) 8 c) 9 d) 10 e) 12</p> |
| 35 | <p>AMC 10A 2018) What is the greatest three digit positive integer “n” for which the sum of the first “n” positive integers is NOT a divisor of the product of the first “n” positive integers?</p> <p>a) 995 b) 996 c) 997 d) 998 e) 999</p> |
| 36 | <p>AMC 10A 2020) A positive integer divisor of $12!$ is chosen at random. The probability that the divisor chosen is a perfect square can be expressed as $\frac{m}{n}$, where “m” and “n” are relatively prime positive integers. What is “m+n”?</p> <p>a) 3 b) 5 c) 12 d) 18 e) 23</p> |

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| 37 | <p>AMC10 2018) For how many (not necessarily positive) integer values of “n” is the value of $4000\left(\frac{2}{5}\right)^n$ an integer?</p> <p>a) 3 b) 4 c) 6 d) 8 e) 9</p> |
| 38 | <p>AMC 10 2014) The largest divisor of 2,014,000,000 is itself. What is the fifth largest divisor?</p> <p>a) 125,875,000 b) 201,400,000 c) 251,750,000 d) 402,800,000 e) 503,500,000</p> |
| 39 | <p>AMC 10 2018) How many of the first 2018 numbers in the sequence are divisible by 101? 101, 1001, 10001, 100001,</p> <p>a) 253 b) 504 c) 505 d) 506 e) 1009</p> |
| 40 | <p>AMC 10 2013) How many three digit numbers are not divisible by 5, have digits that sum to less than 20, and have the first digit equal to the third digit?</p> <p>a) 42 b) 60 c) 66 d) 68 e) 70</p> |
| 41 | <p>AMC 10 2018) Let “a”, “b”, “c”, and “d” be positive integers such that $\gcd(a,b) = 24$, $\gcd(b,c) = 36$, $\gcd(c,d) = 54$, and $70 < \gcd(d,a) < 100$. Which of the following must be a divisor of “a” ?</p> <p>a) 5 b) 7 c) 11 d) 13 e) 17</p> |
| 42 | <p>AMC10A 2019) For how many integers “n” between 1 and 50, inclusive, is $\frac{(n^2-1)!}{(n!)^n}$ an integer. Recall that $0!$ is equal to 1.</p> <p>a) 31 b) 32 c) 33 d) 34 e) 35</p> |

SOLUTION:

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| <p>Problem 1. Solution: D. The sum of the digits must be divisible by 9. $1 + 9 + 8 = 18$ which is divisible by 9.</p> <p>Problem 2. D. 72 is divisible by both 6 and 9 but not divisible by 54.</p> <p>Problem 3. E $nm - (n - 2)(m - 2) = 2(n + m - 2)$ which must be divisible by 2.</p> <p>Problem 4. Solution: E. Solution: Let s be 18 and t be s divided by 21. Then $st = 378$. By the divisibility rule for 9, $3 + 7 + 8 = 18$ which is divisible by 9. So st is divisible by 9. The remainder is 0.</p> | <p>Problem 5. Solution: B. x must be a multiple of 5 so it is divisible by 5. Let us say $x = 10$. $3x = 30$ that is divisible by 15. So we see that both I and III are not true.</p> <p>Problem 6. Solution: D. Since x is divisible by 7 and y is divisible by 8, xy must be divisible by 56. For the same reason, $8x$ is divisible by 56 as well as $7y$. Therefore $8x + 7y$ is divisible by 56.</p> <p>Problem 7. Solution: C. We know that both I and II are true by letting $a = 8$, $b = 4$, $c = 2$, and $d = 1$. We know that III is not true by letting $a = 16$, $b = 4$, $c = 2$, and $d = 1$.</p> <p>Problem 8. Solution: D. (6225, and 6775) If the given number is divisible by 25, $\overline{B5}$ should be divisible by 25. $25 \times 1 = 25$, and $25 \times 3 = 75$.</p> |
| <p>Problem 9. Solution: A. $3 + 1 + d + 2 + 6 = 12 + d \Rightarrow d = 0, 3, 6, \text{ and } 9$.</p> <p>The sum of all possible values of d is $0 + 3 + 6 + 9 = 18$.</p> <p>Problem 10. Solution: D. By the divisibility rule for 7, $6x - 4 \times 2 = 60 + x - 8 = 52 + x \Rightarrow x = 4$.</p> <p>Problem 11. Solution: A. The smallest such number could be $7 + 5 = 12$. $12 \times 5 = 60 \Rightarrow 60 \div 7 = 8 \text{ r } 4$.</p> <p>Problem 12. Solution: E. $3 + n + 8 + 5 + n = 16 + 2n$ is divisible by 3 when $n = 1, 4$, or 7. Since n is even, $n = 4$.</p> | <p>Problem 13. Solution: E. If $5,376,5d4$ is divisible by 24, it must be divisible by both 3 and 8. $5 + 3 + 7 + 6 + 5 + d + 4 = 30 + d \Rightarrow d = 0, 3, 6, \text{ or } 9$. $5d4$ is also divisible by 8. $d = 0$ is the only value that works.</p> <p>Problem 14. Solution: B. There are ninety 2-digit numbers. There are seven 2-digit numbers divisible by 13 with the first one being $13 \times 1 = 13$ and the last one being $13 \times 7 = 91$. The answer is $90 - 7 = 83$.</p> <p>Problem 15. Solution: C. Method 1: The number is divisible by 12. There are 8 such numbers less than 100 with the first one being $12 \times 1 = 12$ and the last one being $12 \times 8 = 96$.</p> |
| <p>Method 2: $\left\lfloor \frac{100}{12} \right\rfloor = 8$. The answer is 8.</p> <p>Problem 16. Solution: E. Let the four-digit number be $abcd$ We have: $a + c = b + d$ $a + c = 5$ and $b + d = 5$.</p> <p>$\Rightarrow a = 4, c = 1, \text{ and } b = 3, d = 2$. We also have to consider the different orderings of the numbers. We get eight numbers that are divisible by 11: 4213, 4312, 1243, 1342, 3421, 3124, 2431, 2134.</p> <p>Problem 17. Solution: D. The given number is already divisible by 2, so we only need to consider the case when it is divisible by 11. Therefore, we must have $d = 2 + 6 = 8$. We see that $286 = 11 \times 26$.</p> | <p>Problem 18. Solution: D. The number formed by last two digits must be divisible by 4. We have the following cases for the last two digits: $\begin{array}{cc} 1 & 2 \\ 2 & 4 \\ 3 & 2 \end{array}$ For all three cases we can produce two such four-digit numbers. The solution is then $6/24 = 1/4$.</p> <p>Problem 19. Solution: D. The last digit should be even and the sum of the digits should be divisible by 3. So the greatest three-digit number is 996.</p> |
| <p>Problem 20. Solution: A. The number should be even and divisible by 9. Since the last digit is already even, we only need to consider the sum of the digits. $d = 4$.</p> <p>Problem 21. Solution: A. The number should be divisible by 3, 5, and 4. Since the last digit is 0, it will be divisible by 5. The sum of the digits should be divisible by 3, so K can be 1, 4, or 7. The last two-digit should be divisible by 4, so the only value for K is 4.</p> <p>Problem 22. Solution: B. If a number is divisible by 4, the last two digits must be divisible by 4. We have the following six cases: We have $3 \times 2 = 6$ ways to fill each two empty spaces. There are $6 \times 6 = 36$ such 4-digit numbers.</p> <p>Problem 23. Solution: E. $72 = 8 \times 9 \Rightarrow m6,79n$ must be divisible by both 8 and 9. When $m6,79n$ is divisible by 8, the three-digit number $79n$ must be divisible by 8 and n must be even. This yields $n = 2$. When $m6,79n$ is divisible by 9, $m + 6 + 7 + 9 + 2 = m + 24$ must be divisible by 9. Note that $0 \leq m \leq 9$, so m must be 3. $m + n = 3 + 2 = 5$.</p> | <p>Problem 24. Solution: D. Method 1 (official solution): The tens digit of a power of 7 is determined by the last two digits of the previous power of 7. The pattern for the last two digits of successive powers of 7 is 01, 07, 49, 43, 01, 07, 49, 43, 01, ... Since $2011 = 4 \times 502 + 3$, the last two digits of 7^{2011} are 43 and the tens digit is 4.</p> <p>Method 2 (our solution) $7^{2011} = (7^2 \times 7^2)^{502} \times 7^3 = [(50 - 1)^2]^{502} \times 343 = (2500 - 100 + 1)^{502} \times 343$ When $(2500 - 100 + 1)^{502} \times 343$ is divided by 100, the remainder is 43. So the answer is D.</p> <p>Method 3 (our solution) Then we have $\begin{array}{ll} 7^{2011} \equiv r & \text{mod } 4 \\ 7^{2011} \equiv r & \text{mod } 25 \end{array} \Rightarrow \begin{array}{ll} 7^{2011} \equiv 3 & \text{mod } 4 \\ (7^4)^{502} \times 7 \equiv r & \text{mod } 25 \end{array} \Rightarrow \begin{array}{ll} 7^{2011} \equiv 3 & \text{mod } 4 \\ (49)^{502} \times 7 \equiv r & \text{mod } 25 \end{array} \Rightarrow \begin{array}{ll} (-1)^{502} \times 7 \equiv r & \text{mod } 25 \\ \Rightarrow -7 \equiv r & \text{mod } 25 \end{array} \Rightarrow \begin{array}{ll} -18 \equiv r & \text{mod } 25 \end{array}$ We try to find the last two digit of 7^{2011}. $\begin{array}{ll} 7^{2011} \equiv r & \text{mod } 100 \end{array}$ (1) can be written as $\begin{array}{ll} (49)^{502} \times 7 \equiv r & \text{mod } 100 \\ \Rightarrow (-1)^{502} \times 7 \equiv r & \text{mod } 100 \\ \Rightarrow 7 \equiv r & \text{mod } 100 \end{array}$ So the last two digit is 43 and the answer is D.</p> |

25) Answer B

26)

Two 7-digit numbers with missing digits must both be divisible by 3. The trick is applying digit-sum rules.

Example (AMC 8 2017 Problem 7):

Problem 7

Let Z be a 6-digit positive integer, such as 247247, whose first three digits are the same as its last three digits taken in the same order. Which of the following numbers must also be a factor of Z ?

(A) 11 (B) 19 (C) 101 (D) 111 (E) 1111

Six-digit numbers of the form $abcabc$ must be divisible by $1001 = 7 \times 11 \times 13$. So 11 is always a factor.

Examples:

Problem 9

What is the sum of the distinct prime integer divisors of 2016?

(A) 9 (B) 12 (C) 16 (D) 49 (E) 63

AMC 8 2016 Problem 9 → distinct prime factors of 2016.

Problem 15

What is the largest power of 2 that is a divisor of $13^4 - 11^4$?

(A) 8 (B) 16 (C) 32 (D) 64 (E) 128

AMC 8 2016 Problem 15 → largest power of 2 dividing $13^4 - 11^4$.

Problem 14

Let N be the greatest five-digit number whose digits have a product of 120. What is the sum of the digits of N ?

(A) 15 (B) 16 (C) 17 (D) 18 (E) 20

AMC 8 2018 Problem 14 → greatest 5-digit number with digit product 120.

Problem 18

How many positive factors does 23,232 have?

(A) 9 (B) 12 (C) 28 (D) 36 (E) 42

AMC 8 2018 Problem 18 → number of factors of 23,232.

Problem 17

How many factors of 2020 have more than 3 factors? (As an example, 12 has 6 factors, namely 1, 2, 3, 4, 6, and 12.)

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Example (AMC 8 2012 Problem 15):

Problem 15

The smallest number greater than 2 that leaves a remainder of 2 when divided by 3, 4, 5, or 6 lies between what numbers?

(A) 40 and 50 (B) 51 and 55 (C) 56 and 60 (D) 61 and 65 (E) 66 and 99

1. 2018 AAMC 8 2016

Problem 24

The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number $PQRST$. The three-digit number PQR is divisible by 4, the three-digit number QRS is divisible by 5, and the three-digit number RST is divisible by 3. What is P ?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Solutions

Solution 1 (Modular Arithmetic)

We see that since QRS is divisible by 5, S must equal either 0 or 5, but it cannot equal 0, so $S = 5$. We notice that since PQR must be even, R must be either 2 or 4. However, when $R = 2$, we see that $T \equiv 2 \pmod{3}$, which cannot happen because 2 and 5 are already used up; so $R = 4$. This gives $T \equiv 3 \pmod{4}$, meaning $T = 3$. Now, we see that Q could be either 1 or 2, but 14 is not divisible by 4, but 24 is. This means that $Q = 2$ and $P = \boxed{\text{(A) 1}}$.

How many 4-digit positive integers (that is, integers between 1000 and 9999, inclusive) having only even digits are divisible by 5?

2. (A) 80 (B) 100 (C) 125 (D) 200 (E) 500

2020 AMC 10B Problem 7

How many positive even multiples of 3 less than 2020 are perfect squares?

3. (A) 7 (B) 8 (C) 9 (D) 10 (E) 12

2019 AMC 10A Problem 9

What is the greatest three-digit positive integer n for which the sum of the first n positive integers is not a divisor of the product of the first n positive integers?

(A) 995 (B) 996 (C) 997 (D) 998 (E) 999

2020 AMC 10A Problem 15

A positive integer divisor of 12 is chosen at random. The probability that the divisor chosen is a

perfect square can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 3 (B) 5 (C) 12 (D) 18 (E) 23

2018 AMC 10A Problem 7

For how many (not necessarily positive) integer values of n is the value of $4000 \cdot \left(\frac{2}{5}\right)^n$ an integer?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 9

2014 AMC 10B Problem 12

The largest divisor of 2,014,000,000 is itself. What is its fifth largest divisor?

(A) 125,875,000 (B) 201,400,000 (C) 251,750,000 (D) 402,800,000 (E) 503,500,000

2018 AMC 10B Problem 13

How many of the first 2018 numbers in the sequence
101, 1001, 10001, 100001, ... are divisible by 101?

(A) 253 (B) 504 (C) 505 (D) 506 (E) 1009

2013 AMC 10A Problem 13

How many three-digit numbers are not divisible by 5, have digits that sum to less than 20, and have the first digit equal to the third digit?

(A) 52 (B) 60 (C) 66 (D) 68 (E) 70

2018 AMC 10A Problem 22

Let a, b, c , and d be positive integers such that $\gcd(a, b) = 24$, $\gcd(b, c) = 36$, $\gcd(c, d) = 54$, and $70 < \gcd(d, a) < 100$. Which of the following must be a divisor of a ?

(A) 5 (B) 7 (C) 11 (D) 13 (E) 17

2019 AMC 10A Problem 25

For how many integers n between 1 and 50, inclusive, is $\frac{(n^2 - 1)!}{(n!)^n}$ an integer? (Recall that $0! = 1$.)

(A) 31 (B) 32 (C) 33 (D) 34 (E) 35

1. MC 8 (Answer B)

The 5-digit number 2 0 1 8 U is divisible by 9. What is the remainder when this number is divided by 8?

(A) 1 (B) 3 (C) 5 (D) 6 (E) 7

Problem 21

The 7-digit numbers 74A52B1 and 326AB4C are each multiples of 3. Which of the following could be the value of C ?

(A) 1 (B) 2 (C) 3 (D) 5 (E) 8

Two 7-digit numbers with missing digits must both be divisible by 3. The trick is applying digit-sum rules.

Why this matters:

